Uplift Quadratic Program in Irish Electricity Price Setting

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Abstract

This report summarises progress made towards the problem submitted by Bord Gáis at the 70th European Study Group with Industry organised by Mathematics Applications and Consortium for Science and Industry (MACSI) and held in the University of Limerick from the 28th June - 3rd July 2009. Bord Gáis required a deeper insight into the dynamics of Uplift prices. Therefore, the aim of the group was to apply a variety of analytical tools to the problem in order to satisfy Bord Gáis requirements. The group conducted a KKT Optimality Analysis of the quadratic program used to determine the Uplift prices, performed statistical analysis to identify the binding constraints and their sensitives to the Uplift prices, simulated a synthetic stochastic process that is consistent with the Uplift pricing series and investigated alternative objective functions for the quadratic program.

Introduction

Energy prices in the Irish Electricity Market are composed of a Shadow Price and an Uplift Component. The Shadow Price is the cost of meeting a marginal unit of demand, as determined by solving the Unit Commitment Problem. The Unit Commitment Problem is a Mixed Integer Program which determines which units (generators) to commit (turn on), so as to minimize the total system cost of generation, subject to meeting demand in each period and respecting technical characteristics of the units (e.g. ramp-rates, min on/off times, minimum generation). Participants declare their true marginal costs and therefore the Shadow Price alone does not guarantee that generators will recover their fixed running costs, e.g. start-up and no-load. The Uplift component of prices addresses this. It is derived via a Quadratic Program (QP) which ensures that the generators recover those costs. The QP aims to minimize a parameterized combination of total additional cost due to Uplift and the volatility of Uplift. The problem involves investigating the quadratic program used to determine the Uplift component of the prices and to examine the behaviour of the this Uplift component. Our aim for the week was to exploit the expertise of the group to develop analytical tools that provided Bord Gáis with a deeper insight into the uplift process.

The report begins by describing the structure of the quadratic program, followed by analysis of the optimal Uplift components using the Karush_Kuhn_Tucker (KKT) conditions. One can also employ statistical analysis to identify the generators contributing to Uplift prices, this method is described and the results produced. The subsequent sections involves simulating a stochastic process that reproduces the Uplift series to a quantitatively comparable level and investigating other objective functions for the QP. Finally, the report concludes with results and suggestions for further research.

1 Problem Formulation

The "System Marginal Price" (SMP), the incremental cost of supplying the next MWh of energy demanded for each half-hourly period h, of day t is

$$\mathsf{SMP}_h = \mathsf{SP}_h + \mathsf{UP}_h.$$

Where SP_h represents the "Shadow prices", the marginal price of electricity per MWhr in each half-hour based on the information provided by the generators and UP_h symbolizes the "Uplift", the correction factor that is applied retrospectively to the shadow prices to cover the fixed running costs.

The SEMO (Single Electricity Market Operator) determines the Uplift function by solving a quadratic program that minimises Uplift revenues (the Cost objective) and minimises the shadow price distortion (the Profile objective)

$$\min_{\mathsf{UP}_h, h=1\ldots 48} \mathsf{F}(\mathsf{UP}_h) \equiv \underbrace{\alpha \sum_h \left[(\mathsf{SP}_h + \mathsf{UP}_h) \sum_g \mathsf{Q}_{gh} \right]}_{\text{Cost objective}} + \underbrace{\beta \sum_h \mathsf{UP}_h^2}_{\text{Profile objective}}, \tag{1.1}$$

subject to,

$$\sum_h [(\mathsf{SP}_h + \mathsf{UP}_h)\mathsf{Q}_{gh}] \geq \mathsf{CR}_g, \qquad \text{for} \quad g = (1, \ldots, G), \tag{1.2}$$

$$\mathsf{UP}_h \geq 0, \qquad \text{for} \quad h = (1, \ldots, 48). \tag{1.3}$$

Given that Q_{gh} denotes the volume (quantity) of electricity provided by each generator g in each half-hour period h and CR_g is the total cost of running for generator g, given by

$$\mathsf{CR}_g = \sum_h [\mathsf{Q}_{gh}\mathsf{C}_u + \mathsf{NLC}_g \mathcal{I}(\mathsf{Q}_{gh} > 0)] + \mathsf{ST}_g, \tag{1.4}$$

Where C_u is the variable (fuel) cost per unit, NLC_g is the no-load cost of generator g, which is the element of operating costs which is invariant with the actual level of output and ST_g is the start-up cost of generator g, the cost of turning on generator g at any time h during day t.

The first constraint given in equation (1.2) ensures that each generator g covers its costs CR_g and the second constraint given in equation (1.3) certifies that all Uplift values are positive.

For convenience equations (1.1,1.2,1.3) were converted into Matlab-like notation as follows;

$$\min_{u \in \mathbb{R}^{48}} \mathsf{F}(u) \equiv \alpha e^T \mathsf{Q}(s + u) + \beta u^T u, \tag{1.5}$$

subject to,

$$\mathsf{Q}(s + u) \geq c, \tag{1.6}$$

$$u \geq 0. \tag{1.7}$$

Please refer to the conversion table in the appendix for the details.

The problem was attacked using four separate but complementary approaches. These approaches all provide a insight into the composition of the Uplift process and our outlined in the subsequent sections. Section one examines the optimality of the quadratic program using the Karush_Kuhn_Tucker(KKT) conditions, section two describes a statistical analysis of both the actual and re-constructed data set, section three develops a stochastic model that mimics the underlying dynamics of the Uplift prices and finally section five investigates the validility of alternative objective functions.

2 KKT Optimality Analysis of the QP

The KKT 1^{st}-order necessary conditions for a general inequality-constrained problem are:

$$\Delta_u \mathcal{L} = 0 \equiv \Delta_u F(u) - \sum_{i \in \mathcal{I}} \lambda_i \Delta_u c_i(u) = 0 \tag{2.1}$$

where $\mathcal{I}$ is the index set for the inequality constraints.

Computing the gradients with respect to u;

$$\alpha Q^T e + 2\beta u - Q^T \lambda_1 - \lambda_2 = 0, \tag{2.2}$$

where $\lambda_{1,g} \geq 0$ for $g = (1, \ldots, G)$ when the corresponding constraints are active and zero otherwise, and $\lambda_{2,h} \geq 0$ for $h = (1, \ldots, 48)$ when the corresponding constraints are active and zero otherwise. As the QP is convex, the KKT conditions are both necessary and sufficient for optimality (i.e the lagrange multipliers are unique). Moreover, the lagrange multiplier λ_2 must be zero for all h. Therefore one can rewrite equation (2.2) as;

$$2\beta u = -\alpha Q^T e + Q^T \lambda_1. \tag{2.3}$$

Under the Single Electricity Market (SEM) trading and settlement code the regulatory authorities determine the weighting parameters α, β in equation (2.2) for each year. The first parameter α governs the importance of the Uplift cost objective, such that $0 \leq \alpha \leq 1$. The second parameter β governs the importance of the Uplift profile objective, such that $0 \leq \beta \leq 1$ and such that $\alpha + \beta = 1$. The regulatory authorities have found that setting the parameters to $\alpha = 0$ and $\beta = 1$ provides the most appropriate balance of cost and price stability. Therefore applying these conditions to equation (2.2), given that several generation constraints represented by $g \in \mathcal{A}_{g_0}$ are binding yields an optimal set of positive uplifts u which are a non-negative linear combination of the columns of Q^T (the rows of Q) corresponding to the binding constraints.

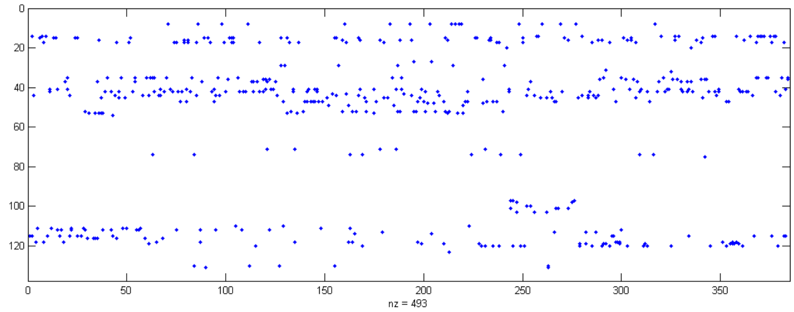
$$2u = \sum_{g \in \mathcal{A}_{g_0}} \lambda_{1,g} \left(q_g \right). \tag{2.4}$$

The binding constraints now take the form $(Qu)_g = c_g - (Qs)_g$ for $g \in \mathcal{A}_{g_0}$.

2.1 Results

Equation (2.4), motivated an analysis of historical data to determine which generation constraints have been binding in each 24-hour period. This is illustrated in figure (1), which clearly shows that the generators have an intrinsic band structure.

Figure 1: Recurrence of Binding Generators



Two small subsets of generators can be identified from figure (1). A set of the same generators that are clearly binding from day to day and a set of generators that are never binding. This provoked a further examination of the classification and frequency of the binding generators. The classification of generators that are binding and therefore influencing Uplift prices are shown in figure (2).

It appears that it is almost always the thermal plants (gas, oil, distillate and coal) that have binding constraints. The frequency of generators that are typically binding is given in figure (3).

It is clear that in general there is only one (45% of all occurrences) or two generators (34% of all occurrences) binding with very few occasions with more than four generators binding (.01% of all occurrences).

Figure 2: Classification of binding generators

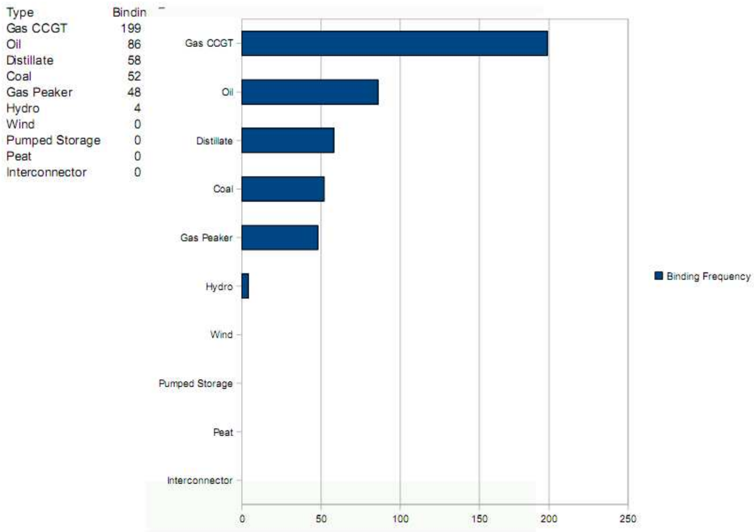


Figure 3: Frequency of binding by generator type

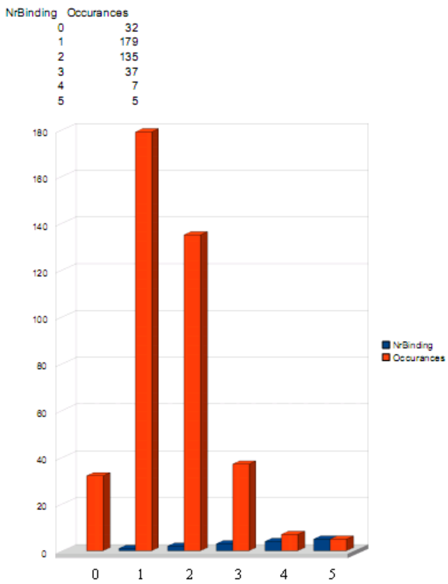


Figure 4: Frequency of binding by generator type

3 Factor Analysis

Using statistical analysis the group tried to identify factors that were contributing to Uplift prices, in order to state the sensitivity of Uplift prices to certain known factors. The Uplift prices are an adjustment price and therefore can have zero values, in this case 40% of the Uplift price values are equal to zero.

A factor model of Uplift prices was employed to decompose the random prices into factor related and specific prices.

$$u = \alpha + \beta_1 f_1 + \ldots + \beta_k f_k + \varepsilon,$$

where u denotes the time series of Uplift prices, α is a single coefficient introduced in order to ensure $E[\varepsilon] = 0$, β_i is the sensitivity of Uplift to the factor f_i and ε represents the error or Uplift specific price's.

If the vector of factors f is known one can model this using a macroeconomic factor model, however if they are unknown and therefore must be estimated one uses a statistical factor model.

3.1 Macroeconomic model

The results from the KKT conditions shown in equation (2.2) are structurally similar to a macroeconomic factor model. The equation states that the factors contributing to Uplift prices u are q_g, the quantity of electricity provided by each generator g in each half-hour period h. Using the macroeconomic factor model, one can verify the results from the KKT conditions with minimal assumptions on the underlying dynamics of the problem. The advantage to using this method is that it can be applied to both the reconstructed Uplift data and the actual Uplift data, producing the binding generation constraints in both cases.

From figures (2) and (3), one can establish that 45% of the generators bind only once, 86% of generators have less than three binding generators and that wind, pumped storage, peat and the interconnector generators don't contribute to Uplift prices. Exploiting this and the fact that all the parameters in equation (2.2) are positive one can identify the binding generators by analyzing the correlation structure between the output of each of the 85 generators that influence Uplift prices and the Uplift prices themselves. The results are shown in figures (5,6).

In figure (5,6), the positive correlation values represented by the light green, yellow, orange and red give the set $\mathcal{A}_g$ of all generators that could be binding in any given day t, for the 385 days considered. From this one can identify how many generators are binding, which generators are binding and the sensitivity of Uplift prices to the binding generators on each day t. For the reconstructed data, there are 76 days in which one generator is binding this corresponds to generators which have a correlation coefficient value greater than or equal to 0.9999. To identify the binding generators when more than one generator is considered one would construct a macroeconomic factor model with the Uplift regressed against all possible generator's identified by the correlation coefficients (> 0 and < 0.9999), the max number of possible binding generators in this case is 42. Therefore one can develop a stepwise algorithm which selects the combination of generators that are binding based on the AIC (Akaike's information criterion) of each combination. AIC makes comparisons between a number of possible models on the basis

Figure 5: Correlations between generator's and Uplift prices for reconstructed data

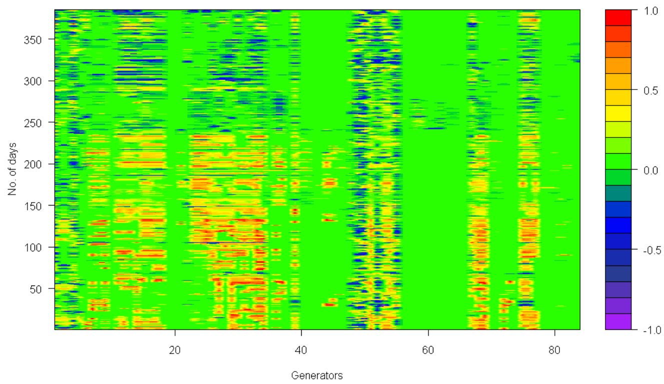
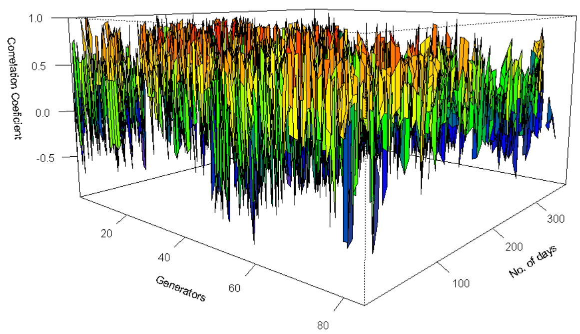


Figure 6: Correlations between generator's and Uplift prices for reconstructed data



of the following statistic: AIC$= -2 \times \log(\text{maximum partial likelihood}) + 2 \times b$, where b is the number of β coefficients in each model under consideration. The AIC is based on maximum likelihood and a penalty for each parameter. The best model has the smallest AIC value. This algorithm produces the binding generators and their sensitivities to Uplift prices on any given day t.

This procedure can also be run on the actual Uplift prices. Thus producing the binding constraints and their sensitivities to actual Uplift prices. Figures (7 and 8) illustrate the results.

There are only 5 days for which only one generator is binding. From figures (5) and (7) one can clearly identify a band structure indicating that there is a recurring group of generators that are binding.

Figure 7: Correlations between generator's and Uplift prices for actual data

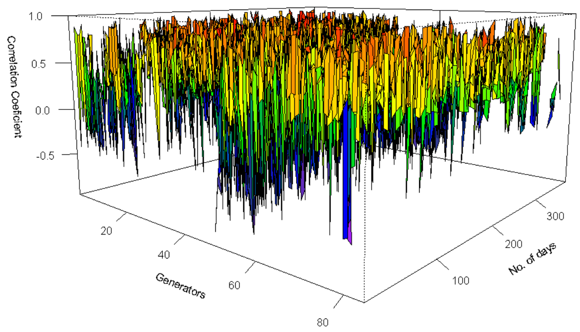
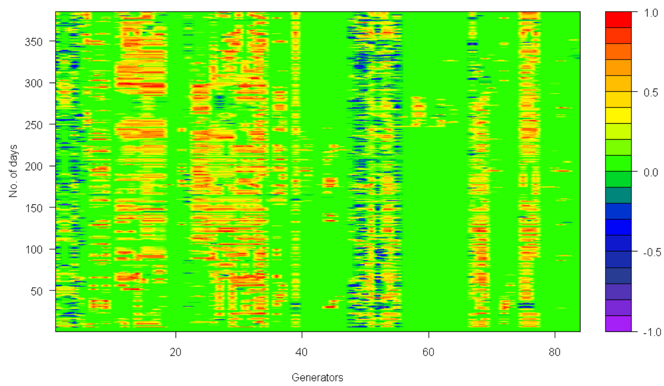


Figure 8: Correlations between generator's and Uplift prices for actual data



4 Uplift as an individual stochastic process

4.1 Motivation for stochastic analysis

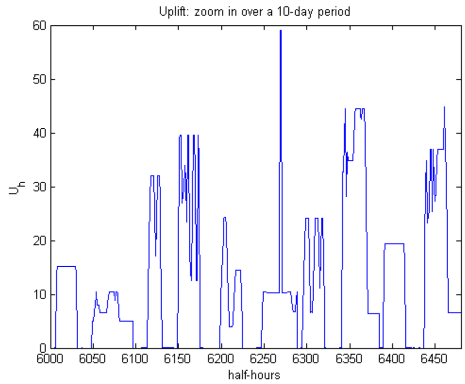
In terms of forecasting, electricity prices are one of the most difficult types of financial time series. This is due to their highly volatile behavior. However, the Uplift prices exhibit some specific statistical features that can be exploited in order to reconstruct the series up to some qualitatively comparable level.

The aim of the group was to reproduce a synthetic stochastic process which would behave similarly to the real Uplift prices, that would be suitable for Monte Carlo simulations or some other stochastic purposes. The following subsections present specific Uplift features that were identified and employed in simulations to generate the synthetic process and an examination of the synthetic process produced.

4.2 Identified features of the Real Uplift process

Initially the group examined the path of the Uplift data over consecutive days. This is illustrated in figure (9). One can clearly identify a jump structure inherent in the data.

Figure 9: Path covering a few consecutive days of Uplift half-hourly data



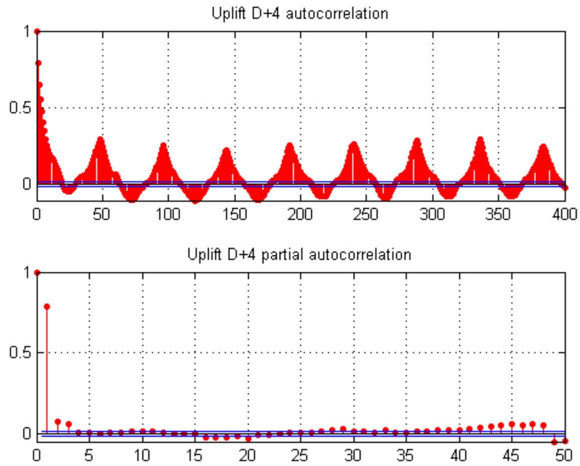
Considering that by definition Uplift must be positive for all time t and the length of the plateaus may be similar to waiting times. One can assume a Poisson distribution to model the lengths of plateaus in the Uplift prices and an investigation of the empirical Uplift jump distributions could be used to verify the jump heights.

Since electricity is highly dependant on half-hourly demand (contributing to half-hourly, daily and annual seasonality) it is reasonable to expect the Uplift prices to also reflect some seasonality features. These features can be clearly identified by the autocorrelation function (ACF) and partial autocorrelation function (PACF) for the Uplift price series presented in figure (10). As desired, we clearly see peaks in ACF repeating with $48-$lag regularity. Additionally, they are slightly locally maximal points for every 336^{th} lag (48 half-hours times 7 week days), portraying weekly seasonality but this is not as significant as the daily seasonality. Thus, in the simulation, in order to reproduce the daily seasonality the current observation is regressed against the observation that occurred 48 half-hour periods earlier. One would suspect that there is also an annual seasonality unfortunately there is an insufficient amount of data to verify this claim.

4.3 Uplift jump waiting times

A Poisson distribution was employed to model jump waiting times in the Uplift process. However, due to time limitations, the parameters were set using heuristic methods. It is evident from Figure (9), that waiting times are not distributed uniformly, as there is a clear dependency on the current price level. In particular, the process zero level components are considerably long, whereas at high levels (40-100), the process jumps down almost immediately. Therefore,

Figure 10: Autocorrelation and partial autocorrelation functions for the Uplift price series.



it seems sensible to set four different price levels, defined by various values of the Poisson λ parameter. These values are as follows;

- $\lambda = 50$ for $u_h = 0$.
- $\lambda = 10$ for $0 < u_h \leq 40$.
- $\lambda = 2$ for $40 < u_h \leq 100$.
- $\lambda = 0.05$ for $u_h > 100$.

4.4 Jump up/down probability and Jump heights

One can also clearly identify a jump structure in figure (9). The higher the value of current Uplift, the higher the probability of a negative jump. Another trait that becomes apparent from examining the data and figure (9) is that if the current price level crosses some considerably high price level (in this case $u_h > 50$), then the process exhibits continuous negative jumps until it reaches zero. Sometimes, these negative jumps occur in a few time steps but often within one time step only.

Two empirical cumulative distribution functions conveying jump direction are derived from the occurrences of positive and negative movements in the Uplift series for the purpose of random number generations. Figure (11) is a histogram of the occurrences of jumps up and down for a given Uplift level. The histograms structure conveys an exponential curve.

The distribution of the jump magnitude conditional on the price level is derived by computing the density function of the jump magnitudes. This is illustrated in figure (12).

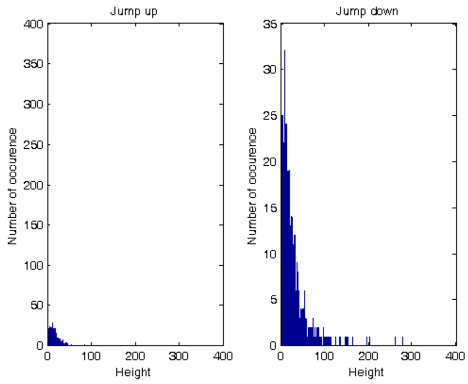


Figure 11: Histograms for occurrences of jumps up/down depending on current Uplift level.

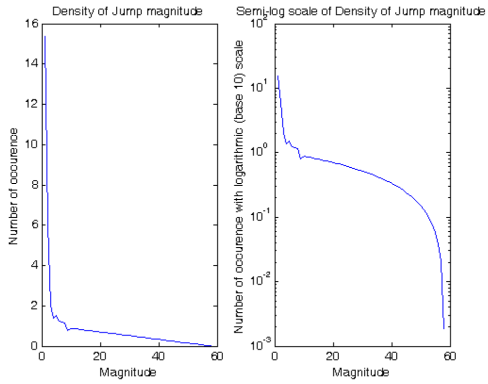


Figure 12: Density function of the jump magnitudes.

4.5 Plateaus within night hours

Plateaus during the night half-hour intervals are also apparent in the data these portray a high probability of constant (usually zero) Uplift during this period. Once the base process was simulated one could now induce this attribute by setting the night half-hour intervals equal to zero, based on a uniformly distributed probability of zero level continuing for 12 to 20 half-hour periods.

4.6 Algorithm

The general algorithm for the simulation approach is as follows;

1. For $h = (1, \ldots, 48)$ set $u_{a,h} = u_{s,h}$. Where $u_{a,h}$ is the actual Uplift price for the half-hour interval h and $u_{s,h}$ is the simulated Uplift price for the half-hour interval h.

2. Given $u_{s,h}$ for a particular half hour h, identify the corresponding Poisson parameter.

Using this parameter generate the length l of the plateaus (amount of half hour intervals h for which the process remains constant).

3. Set the current time point h_c, to the previous time point h_p plus the length of the plateau l. $h_c = h_p + l$.

4. For $l > 1$ then $h = (h_p + 1, \dots, h_p + l)$ let $u_{s,h} = u_{s,h-1}$ and for $l = 1$, then $h = (h_p + 1)$ let $u_{s,h} = u_{s,h-1}$.

5. Then, Given that $u_{s,h}$ is the last price value after the last jump.

   - Set a limiting threshold T. If $u_{s,h} > T$, force the process into negative jumps until $u_{s,h} = 0$.
   - If $u_{s,h} = 0$, force the process to only allow positive jump's generated from the empirical distribution.
   - If $0 < u_{s,h} < T$, determine from the empirical distribution of jump direction whether the process jumps up or down, and then based on the empirical distribution of jump magnitude determine the magnitude of the jump. Sum the sampled jump magnitude and the regressed price value that occurred 48 periods ago. Set $h_c = h_c + 1$.

6. Repeat until the h_c exceeds the target process length to be simulated.

4.7 Results

The simulation was implemented to construct a sample of n half-hour intervals of the Uplift series. Figure (13) presents the simulated Uplift series which is quantitatively (by distribution parameters) comparable with the original Uplift series. Moreover, it is clear that our simulation is also able to produce values significantly differing from the process mean, which is also reflected in the real data.

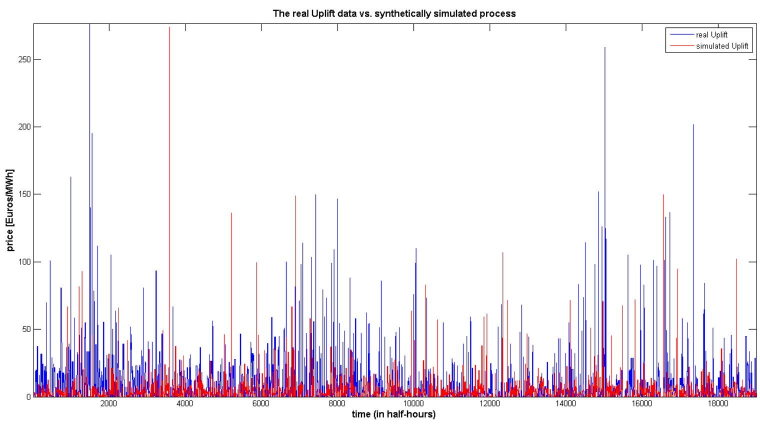
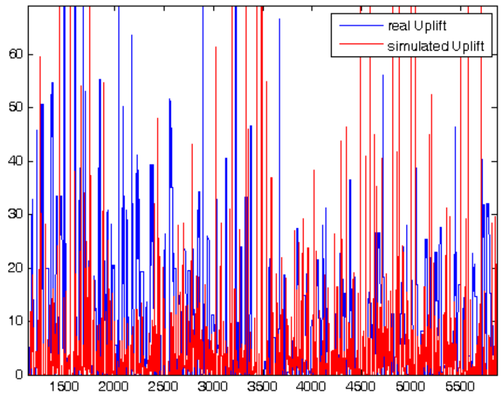


Figure 13: The real and simulated Uplift process.

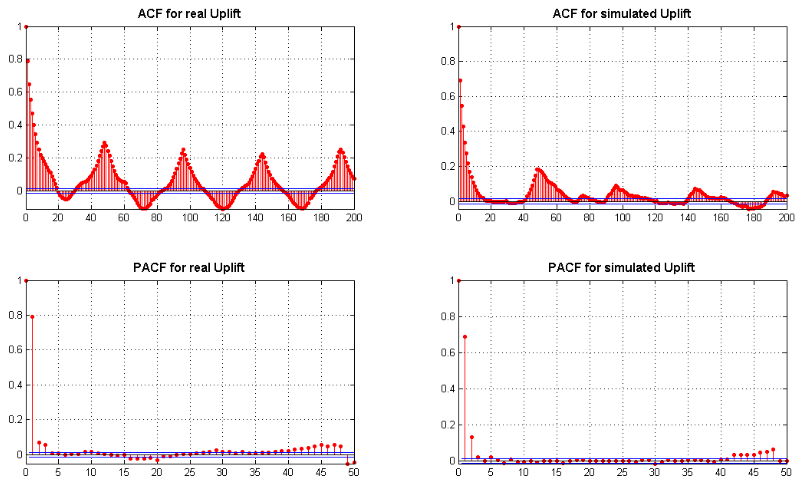
In order to identify if the structural elements of the simulated Uplift process mimic those evident in the actual process, one would compare the two data sets microstructure. This is illustrated in figure (14). The figure confirms that the jump structure is accurately modelled by the simulated process and that the plateaus (more persistent when price at zero level) occur with different lengths at different price levels in both the actual and simulated data.

Figure 14: The real and simulated Uplift process.



Finally, figure (15) illustrates the ACF and PACF of the actual and simulated Uplift data. The half-hourly seasonality, apparent in the actual data has been reconstructed in the simulated data up to a significant level. Additionally, the PACF's of both the original and simulated data have similar features.

Figure 15: Autocorrelation and partial autocorrelation functions of the real and simulated Uplift process.



5 Other Objectives

The group noticed that the objective formula equation (1.5) is not dimensionally consistent, the cost objective function has units of money whereas the profile objective function has units of money per load by time, all squared. This is dissatisfying, it means for example the currency has to specified as well as α and β and it has the practical problem that there is no reason to expect the cost objective and the profile objective to have a similar scale in fact, for the uplifts calculated with the $\beta = 1$ objective

$$\begin{aligned} \bar{C} &= 684651 \\ \bar{P} &= 13478. \end{aligned} \tag{5.1}$$

Where $\bar{C}$ is the mean of the cost objective and $\bar{P}$ is the mean of the profile objective, taken over all the days considered. Of course, even if α and β were fine tuned to make these similar, the changes as UP_h varies would not be comparable.

In order to solve this problem of inconsistency one would have to identify an objective function that explicitly referred to total cost and to variance so that the balance between the two could be decided more transparently as a matter of policy by the industry. Hopefully this would also allow the actual economic cost of reducing variance to be computed as the mixture of these two terms was varied.

To do this in a dimensionally consistent way seems to require that the same method of averaging is used for both terms, at the moment C the cost objective effectively averages over load while P the profile objective averages over time; for a given quantity x_t, lets use

$$\langle x_t \rangle_t = \frac{1}{48} \sum_t x_t \tag{5.2}$$

for the time average and

$$\langle x_t \rangle_Q = \frac{\sum_{gt} Q_{gt} x_t}{\sum_{gt} Q_{gt}} \tag{5.3}$$

for the load average; at the moment

$$\begin{aligned} C &\approx \langle u_t \rangle_Q \\ P &\approx \langle u_t^2 \rangle_t \end{aligned} \tag{5.4}$$

There are, correspondingly, two consistent objectives which would separate out cost and variance

$$E_t^2 = \mu \langle u_t \rangle_t^2 + (1-\mu)\left(\langle u_t^2 \rangle_t - \langle u_t \rangle_t^2\right) \tag{5.5}$$

and

$$E_Q^2 = \mu \langle u_t \rangle_Q^2 + (1-\mu)\left(\langle u_t^2 \rangle_Q - \langle u_t \rangle_Q^2\right) \tag{5.6}$$

Both are quadratic objectives, both are dimensionally consistent and both have separate variance and cost terms.

Another approach to the objective is to decide that the Uplift price should be chosen so that the time course of the Uplift reflects the actual time course of the cost of generation; this may have the advantage of communicating economic signals to the consumers about the actual half-hour by half-hour cost of producing electricity. In short, imagine there wasn't a single Uplift, but instead a time-constant Uplift u_g for each generator period. This would spread the economic cost of g's electrical production evenly across g's load:

$$u_g = \frac{c_g}{\sum_t Q_{gt}} \tag{5.7}$$

The most obvious way to use this generator-period Uplift to calculate u_t would be to take the maximum u_t at every half-hour. This has the aesthetic advantage of matching this method to the method used to calculate the shadow price; it is however very expensive, it has an average cost of

$$\bar{C} = 2690010 \tag{5.8}$$

An alternative would be to calculate a load weighted average of the u_g:

$$\tilde{u}_t = \frac{\sum_g Q_{gt} u_g}{\sum_g Q_{gt}} \tag{5.9}$$

This Uplift would be very cheap;

$$\bar{C} = 72627 \tag{5.10}$$

and would give an Uplift time course well matched to the time course of the electricity generation cost, however, it does not satisfy constraints. The idea, therefore, would be to use this to pose an objective

$$\tilde{E} = \sum_t (\tilde{u}_t - u_t)^2 \tag{5.11}$$

Other profiling goals could be posed; for example related to the shadow price.

Similarly, it might be possible to find a linear objective; C is already linear, all that would be required would be a linear approach to imposing the profiling goal. One method would be to specify a restricted space for u_t; it could be required that it tracks the total load, or the loads of the relevant generator periods or the shadow price or some positive weighted combination of these.

5.1 Conclusion

This report uses various modelling approaches to gain a deeper insight into the underlying dynamics of the Uplift prices. Initially, the group examined the optimality of the quadratic program used to derive the Uplift prices using the KKT conditions. This resulted in equation (2.4) which states that the optimal set of positive uplifts u are a non-negative linear combination of the columns of Q^T (the rows of Q), corresponding to the binding constraints. This

result motivated an analysis of historical data to determine which generation constraints have been binding in each 24-hour period. In order to identify these binding generation constraints one can compute brute force searches or employ statistical analysis. The statistical analysis in section (3) exploits the results computed from section (2) and utilizes correlation coefficients and factor analysis to identify the binding generation constraints for each day $\mathtt{t}$ and the sensitives of Uplift prices to these binding generation constraints. While the statistical and brute force searches are both very separate approaches, the results from both methods were equivalent for the reconstructed data. Surprising results were that there is only a small amount of binding generators and that hydro plants were contributing to Uplift prices. Both methods also identified a band structure to the binding generation constraints, implying that there is a recurring group of generators that are binding. The advantage of using the statistical analysis is that the algorithm can be applied to the actual Uplift prices, producing the binding generation constraints and the sensitives of actual Uplift prices to these binding generation constraints. The group then developed a synthetic stochastic process which behaves similarly to the Uplift pricing series. By identifying specific statistical features of the Uplift prices the group was able to simulate a stochastic process which is qualitatively comparable with the actual Uplift prices, conveys structural elements that are inherent in the actual Uplift prices and portrays the half-hourly seasonality evident in the actual Uplift prices. Finally, the group investigated possible alternative objective functions for the quadratic program used to compute Uplift prices as the objective function used by the SEMO is dimensionally inconsistent. Alternative objectives were posed and discussed.
Propositions for further research; a statistical factor analysis, where the parameters of interest are proxies which are influencing Uplift prices such as Oil prices, weather conditions etc, an investigation into the parameters of the Poisson distribution for modelling the waiting times and the validation of a linear profiling objective. A distributional analysis of Uplift for each day $\mathtt{t}$ and an examination of the effect of annual seasonality with the aid of extra data can be found in ([3]).

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Appendix

Notation In industry standard notation the QP is:

$$\min_{UP_h, h=1...48} F(UP_h) \equiv \alpha \sum_h \left[(SP_h + UP_h) \sum_g Q_{gh} \right] + \beta \sum_h UP_h^2 \tag{5.12}$$

subject to

$$\sum_h [(SP_h + UP_h) Q_{gh}] \geq CR_g, \text{for } g = 1 \ldots G \tag{5.13}$$

$$UP_h \geq 0, \text{for } h = 1 \ldots 48 \tag{5.14}$$

Here the range of h is $h = 1 \ldots 48$ and the range of g is $g = 1 \ldots G$.

Writing the above in a more abbreviated notation where:

$$u = (u_1, \ldots, u_{48}), u_h \equiv UP_h, h = 1 \ldots 48$$
$$s = (s_1, \ldots, s_{48}), s_h \equiv SP_h, h = 1 \ldots 48$$
$$c = (c_1, \ldots, c_G), c_g \equiv CR_g, g = 1 \ldots G$$

we have

$$\min_{u_h, h=1...48} F(u_h) \equiv \alpha \sum_h \left[(s_h + u_h) \sum_g Q_{gh} \right] + \beta \sum_h u_h^2 \tag{5.15}$$

subject to

$$\sum_{h} [(s_h + u_h)Q_{uh}] \geq c_g, \text{for } g = 1 \ldots G \tag{5.16}$$

$$u_h \geq 0, \text{for } h = 1 \ldots 48 \tag{5.17}$$

The above can be written in a Matlab-like notation as

$$\min_{u \in \mathbb{R}^{48}} F(u) \equiv \alpha e^T Q(s+u) + \beta u^T u \tag{5.18}$$

subject to

$$Q(s+u) \geq c \tag{5.19}$$

$$u \geq 0 \tag{5.20}$$

where e is a column vector of size G of ones.