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An Investigation into the Physics of Blowing Polysilicon Fuses

brought by *Analog Devices*,

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Summary: The semi-conductor fuses in this research are fabricated on a submicron process. A voltage potential is applied across the fuse, in order to achieve a blow. This current peaks with a short pulse in the order of tens of milliamps which has a long decrease to zero current flow, resulting in a blown fuse. A fuse blows due to the pinching together of electrically insulating material which initially surrounds the conducting pathway. The pinch cuts across the conductor, and so halts the current flow. In small-geometry fuses a cavity also forms during the blowing process.

The company wishes to understand the fuse blow process mathematically in order to develop a model that can accurately simulate the blowing of the fuses. This report records the thermal, electrical, solid and fluid mechanics of the blowing process that was discussed at the Study Group, with remarks on possible future research for modelling the process.

1. Introduction:

Electrically blown On Board Polysilicon Fuses (OBPF) are used in integrated circuits as one time programmable memory bits. A polysilicon fuse is blown by placing a high enough voltage potential across it. The ensuing current flow results in the fuse blowing to a high resistance state. The fuses must be stable and reliable over years of use, in the sense that (i) when a fuse blows it does so because preset threshold conditions have been exceeded and (ii) that after the fuse has blown its resistance remains stable.

The success of a blow can be measured indirectly by monitoring the current $I(t)$ through the fuse as a function of time t after the voltage is first applied. See figure 1. The current rises very rapidly to its peak. The characteristics of this $I(t)$ curve indicate the reliability of the blow achieved. A peak in the current of tens of milliamps, followed by a longer tail towards zero current flow, is indicative of a good blow for a poly fuse.

These timescales and the shape of this current pulse curve are useful for judging which physical processes may be chiefly responsible for a successful blow. The company have fully characterised these fuses and have determined experimentally the optimum conditions necessary for reliably blowing the fuses. The company wished to know whether the relevant physics of the blowing process could be elucidated, and described mathematically, in terms of the temperature dependent physical properties of the electrically conducting and insulating materials used. We adopt the convention of the +6 volts potential being applied at the bottom (left) of figure 2 , zero volts at the top (right), so that electrons flow down (leftward) and the small fuse blows at the top (right), where the cavity (when it appears) also occurs.

2. Physics of the Fuse-Blowing Process:

A current $I = 90$ milliamps passes along the fuse. The current is carried by electrons (without any counterflow of positive ions). The electrons move in the electric potential gradient induced by the difference of V volts across the fuse (we took $V = 6$ volts). For a fuse whose cross-sectional area (in the plane normal to the current direction) is A, the average current density (in amps per metre-squared) is $J = I/A$. This is enormous in such a small cross-sectional area: $A = 0.35 \times 0.35 \times 10^{-12}\text{m}^2$. Here $A = 10^{-13}\text{m}^2$ to one significant digit, so $J = 90 \times 10^{-3}/10^{-13} = 10^{12}\text{amps}\,\text{m}^{-2}$. Such a large J gives rises to Joule (Ohmic) heating at a rate Q watts m^{-3} given by $Q = \sigma E^2$ where σ is the specific electrical conductivity, and, for a fuse of length L in metres, the electric field strength is $E = V/L = 6/(1.5 \times 10^{-6}) = 4 \times 10^6$ volts per metre. The conductivity σ increases with temperature because more electrons are liberated from the semiconductor at higher temperatures. A model for this is $\sigma = \sigma_0 + \sigma_1 T$, where T is the temperature above the room temperature, and σ_0 and σ_1 are positive constants.

The quantity Q is a mathematical source term in the partial differential equation of heat diffusion which governs the temperature T:

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + (\rho c_v)^{-1}Q(T) \quad . \tag{1}$$

where c_v is the specific heat capacity of the conductor of density ρ, D is the (assumed constant) thermal diffusivity (=thermal conductivity $\times\rho^{-1}\times {c_v}^{-1}$) of the conducting solid, and Q is a positive and increasing function of T. To emphasize the dependence of Q on T in equation (1) we have written $Q(T)$. Equation (1) immediately suggests the possibility of a solution $T(t)$, such that the diffusive terms $D\nabla^2 T$ are together zero. In this case thermal runaway to high (melting) temperatures can occur. The calculation shows that the Joule heating Q is big enough to melt, within nanoseconds, the masses of solid conducting materials of tungsten-silicide and polysilicon, at a temperature somewhat greater than 1127°C. The heat generated in the small fuse is capable of melting a cube of conducting material of side one micron – five times the volume of the fuse. (The polysilicon melts at 1127°C and tungsten silicide melts at 2041°C, but the conducting mixture's melting temperature is substantially reduced by eutectic.)

Aside: A non-dimensional measure of the ability of a region to melt due to the available heat supply is the Stefan number, $St = c_p T_d / L_a$, where c_p is the specific heat capacity at constant pressure, T_d is the temperature difference between phases, and L_a is the latent heat of melting. For this problem $St = 0.1$ which is small enough for us to be confident that the solid mass melts directly into a liquid, without the presence of a so-called "mushy layer" mixture of the two phases.

The conducting tungsten silicide and polysilicon materials shrink when they melt – this anomalous behaviour is shared by ice melting to water. (Solid silicon has a density of 2329kg m^{-3}; as it melts and shrinks its density increases to 2570kg m^{-3}.) On melting we immediately have a relatively low-pressure hot liquid core which (for these small fuses) also means some melting of the two end zones of the conductor, where the fuse connects with the rest of the circuitry. Where the connectors broaden at the ends of the dog bone, the current density is lower because the cross-sectional area of conducting pathway is larger. The image in figure 2 shows melting of the whole region.

The melted material next does several things, which we will treat below in the order of their occurrence:

2.1: First the hot liquid is still electrically conducting (the conductivity increases with temperature) and it is therefore still able to carry the high electron drift force. This force moves the electrons. The force is exerted on the ions in the solid lattice due to the collisions by electrons. The electrons cannot displace the solid matter of the unblown fuse, but it can easily move the liquid while the fuse is in a melted state.

2.2: The hot melted core of the fuse melts the inner layer of the surrounding, electrically-insulating silicon dioxide, which has a higher melting temperature (1427°C) than the tungsten silicide and polysilcon conductor (about 1127°C). The melted core has low dynamic viscosity ($\mu = 5 \times 10^{-4}\text{kg m}^{-1}\text{s}^{-1}$ which is runnier than water); the melted insulator is a thin layer of high dynamic viscosity ($\mu = 10\text{kg m}^{-1}\text{s}^{-1}$, which is more viscous than cold honey). Over a time t the thickness d of melted insulator is $d = (Dt)^{1/2}$, where $D = \kappa/(\rho c_v) = 10^{-7}\text{m}^2\text{s}^{-1}$ is the thermal diffusivity of the insulator, and κ is its thermal conductivity. Over 40 nanosecond (the time to melt the fuse) we expect $d = 0.06$ micron, which is thin compared with the cross-sectional width of the fuse, so we are right to think of the melted insulator as a thin viscous coat around the broad runny melted core. Further away the rest of the insulator is still solid and appears undisturbed by the blow, in photographs of sections of blown fuses.

2.3: The low pressure of all the melt pulls on the viscous coat around it and pulls on the solid insulator beyond. This pulling may cause the longest displacements at the four corners of the small fuse. Lobes of matter appear, in the photos, to have invaded the region of the conductor. Where this displacement pinches off the conducting material, the fuse is no longer able to conduct current. In longer fuses this pinching occurs at the central section of the fuse, and there is no cavity. In the smaller fuses the pinching occurs most at the top end of the fuse (the end which has the lower electrical potential).

2.4: The pulling by the low-pressure melt on its solid surroundings can also create a flow in the viscous coating, and the coat may most easily flow from one corner of the insulator at the fuse's ends, but we interpret the photographs as showing the conducting liquid being able to pull out a nose from the wall in the central section of the longer type of fuse. The nose, made of insulating material, is pulled inward from all around the coat, and so forces the conducting material to form a narrowing neck which is being strangled by the enclosing insulator. As the conductor's cross-sectional area shrinks, the electrical resistance goes up, and the total current that can pass along the fuse goes down, as measured (see figure 1). This can be used as a test of the theory. If the insulator completely strangles and pinches off the conductor, then the insulator blocks the electrical path and the fuse has successfully blown.

2.5: The electron drift force promotes downward flow in the core: this may also lower the pressure in the narrowing gap during strangling. The force per unit volume was found above to be $F_0 = 10^{18}\mathrm{Nm}^{-3}$. So the total force on the fuse is $F_f = 10^{18} \times 1.5 \times 10^{-6} \times (0.35 \times 10^{-6})^2 = 0.15\mathrm{N}$. This force acts on a mass $m = 4 \times 10^{-16}\mathrm{kg}$. This suggests the liquid acceleration downward is $g = F_l/m = 4{\times}10^{14}\mathrm{ms}^{-2}$. In a time of 40 nanoseconds, this gives an unrestrained displacement, from rest, of $X_l = 0.5gt^2 = 0.32\mathrm{m}$. Of course the flow is restrained, by the solid boundary surfaces, but the calculation shows that if any cavity will form it will be at the other end from that to which the liquid flows. There is also plenty of force available to move the viscous fluid on the walls. In terms of a pressure difference $p_d = F_0 L = 10^{18} \times 1.5 \times 10^{-6} = 1.5 \times 10^{12}\mathrm{Nm}^{-2}$

2.6: There is vigorous mixing of the three materials of the fuse, as seen in the debris in photographs. On solidification, this mixture of particles seems to occupy a smaller volume than before they melted, because we see a cavity in small blown fuses. (No cavity appears in larger fuses.) It is unknown if the mixture is in a compressed state after the fuse has blown. But this seems unlikely, as very large stresses of magnitude $\mathrm{E}\Delta\rho/\rho$, would occur where E is Young's modulus, and $\Delta\rho/\rho = 0.1$ is the relative change of volume. Such a change (to an elastically compressed post-blown state), might account for the size of cavity seen, which occupies perhaps 10% of all the volume of space occupied by disturbed matter seen in the photographs. The conducting materials shrink by 10% during melting but this does not explain the cavity *after re-solidification*. We can identify this with a pressure used below $p_0 = \mathrm{E}\Delta\rho/\rho = 10^9\mathrm{N\,m}^{-2}$.

2.7: The 10% by volume of shrinkage of the liquid liberates bubbles or cavities within the liquid, and these bubbles also react to the electron drift force in the same way that a gas bubble has a buoyancy force exerted on it, in response to gravity. Any bubble motion is subject to fluid resistance forces. A surface-energy production argument suggests that many small bubbles can be made, not just one growing bubble/cavity. Suppose a typical bubble has radius r smaller than one tenth say of the width of the fuse, 10^{-8}m. The bubble is acted upon by two forces: the electron drift body force $F = F_0 4/3\pi r^3$, where the force per unit volume $F_0 = nqE$, where there are n electrons per cubic metre, each of charge $q = -1.6 \times 10^{-19}$ coulombs. Here $F_0 = 10^{29} \times 1.6 \times 10^{-19} 6/1.5 \times 10^{-6} = 10^{18}\mathrm{Nm}^{-3}$. The second force on the bubble is necessarily equal and opposite, and is the hydrodynamic drag due to the relative speed U of the bubble with respect to the surrounding liquid, which is also moving. The core flow has a high Reynolds number. So this second force has magnitude $F_l = 1/2\rho C_d \pi r^2 U^2$, where the drag coefficient C_d is about 1 for a sphere.

This gives a bubble speed of $U = (8F_0 r/[3C_d\rho])^{1/2} = 4 \times 10^3 \mathrm{ms}^{-1}$, or slower for bubbles with radius less than 10^{-8}m. The displacement of the bubble with respect to the fluid, over time t is $X = Ut$ so that over 40 nanoseconds $X = 10^3 \times 4 \times 10^{-8} = 40$microns. So even the smallest (and slowest) bubbles may have time to travel the whole length of the fuse and collect in a notional cavity. In a better estimate we would lower U (by a factor one-half?) to account for the liquid's displacement downwards while the bubble is rising.

2.8: The cavity forms at the top of the blown fuse. We do not know whether the cavity grows alone from one nucleation point, or is formed from the gathering together of many bubbles which have moved up to the top of the fuse, propelled there by a buoyancy-type reaction to the electron force. In the small fuses the cavity forms above the point of pinchoff. There is sufficient excess thermal energy to create new internal liquid surfaces for many bubbles, but we did not take this further in the discussions. Further work needs to be done on the upper part of the cavity's surface where the boundary material could be either insulator (a second pinchoff) or conductor. If pinchoff is not achieved, but the cavity blocks the conducting pathway, then conducting material at the top and bottom surface of the cavity may be present, and this would be a situation in which needle re-connection could potentially occur.

3. Mathematical Modelling of Fuse Blowing:

3(a) Pinch-off model:

The full equations of fluid motion are the Navier-Stokes equations for viscous flow, with an equation of mass continuity, and an equation of state relating pressure to density. This model isolates those balancing terms in the Navier-Stokes equations which describe the viscous stress in the conductor and the pressure gradient induced by the fluid when it melts and contracts. We are left with a so-called Stokes flow, particularly appropriate for the low Reynolds number flow of the melted insulator:

$$\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2} + \frac{\partial^2 \mathbf{u}}{\partial z^2} = \mu^{-1} \nabla p \quad , \tag{2}$$

where $\mathbf{u}$ is the vector field of fluid velocity and p is the pressure field. It is likely that in the growing nose $\mathbf{u}$ is mostly in one direction (along the x-axis). Also on the left-hand side of (2) we can configure the coordinates such that the z-derivative is much larger than the x- or y-derivatives, rather like in Hele-Shaw flow where the z-axis is in the direction normal to the parallel plates of a Hele-Shaw cell. If so then we can integrate in y, z space with $u = 0$ at the top and bottom of the fuse, in accord with the idea that the nose lengthens by moving fluid mainly along its own centreline and does so with fluid being drawn along the length of the fuse from the thin melted layer next to the wall. Doing this integration we obtain a model field equation

$$u = -\frac{d^2}{2\mu} \frac{\partial p}{\partial x} \quad , \tag{3}$$

where the constant d is a length scale associated with the fluid domain of the nose of viscous liquid drawn off the wall, say $d = 0.06$ micron at most, but the magnitude of d remains in debate. We have not considered mass continuity equation ($\nabla.\mathbf{u} = 0$ for an incompressible fluid) but we suppose that the other, negligible, velocity components (in the y- and z-directions) can supply the flow of fluid into the domain of the model discussed below.

Suppose that we model the pinchoff as occurring along the x-axis, drawn across the fuse, normal to the current flow. Viscous fluid (SiO_2) of viscosity $\mu_1 = 10\text{kg m}^{-1}\text{s}^{-1}$ lies in a region $x : 0 < x < s(t)$ and a lower viscosity fluid (polysilicon and tungsten silicide) of viscosity $\mu_2 = 5 \times 10^{-4}\text{kg m}^{-1}\text{s}^{-1}$ lies in the region $x : s(t) < x < w$. Here the constant $w = 0.5 \times 0.35 \times 10^{-6} = 1.75 \times 10^{-7}\text{m}$ is half of the width of the fuse. The quantity to be found is the position $x = s(t)$ of the moving interface boundary between the two fluids – it's the tip of the advancing nose. If the nose reaches the centreline at $s = w$ then its mirror-image-nose will have advanced from the right in the opposite direction too, so that the two noses meet and together they pinch off the current. We want to know the amount of closure as a function of time (to compare with the measurements of current) and how long the pinchoff takes.

Let the x-component of the velocity be u, which depends on x and t so we write $u(x, t)$. Because there are two fluids we use subscripts to distinguish variables in the left-hand (sticky) region (subscript 1) from the right-hand (runny) flow region (subscript 2). In the left-hand fluid $u = u_1$ and the pressure $p = p_1(x, t)$ and in the right-hand fluid we call $u = u_2(x, t)$ and $p = p_2(x, t)$. We know that at $x = 0$, which is the solid boundary of the insulator, $u_1 = 0$. Also at the right-hand end, by symmetry of the flow, $u_2 = 0$ at $x = w$. So all we need is linear variation of u with respect to x, and (due to eq. (3)), we only need p_1 and p_2 to depend quadratically on x. Also the partial x-derivative of pressure must vanish at $x = 0$ and at $x = w$ to be consistent with equation (3) at the boundaries.

Using all the information in the above paragraph gives some motivation for just writing down the following expressions

$$p_1(x) = p_0 - \lambda_1(t)x^2 \tag{4}$$

$$p_2(x, t) = p_w + \lambda_2(t)(w - x)^2 \tag{5}$$

so that equation (3) implies

$$u_1(x, t) = \frac{d^2}{\mu_1}\lambda_1(t)x \tag{6}$$

$$u_2(x, t) = \frac{d^2}{\mu_2}\lambda_2(t)(w - x) \quad . \tag{7}$$

In eq. (4) we have a constant pressure p_0 which we can identify with (?) the stress induced by the liquid shrinkage $p_0 = 10^9\text{Nm}^{-2}$ at $x = 0$. In eq. (5) we have a constant pressure at $x = w$ which we do not know, but the melting is likely to make the pressure there so low that we will take the value $p_w = 0$, i.e. absolute zero pressure. In the model we have three time-dependent unknowns $\lambda_1(t), \lambda_2(t)$ and a still unknown position for the fluids' interface at $x = s(t)$. There are three conditions not yet satisfied: they are a dynamic condition, that the pressure is continuous across the interface ($p_1 = p_2$ at $x = s(t)$), and two further kinematic conditions that the velocity is continuous and equal to the speed of the interface ($u_1 = u_2 = ds/dt$ at $x = s(t)$).

Solving these algebraic equations in terms of s gives:

$$\lambda_1(t) = \frac{\mu_1}{sd^2}\frac{ds}{dt} \quad , \tag{8}$$

$$\lambda_2(t) = \frac{\mu_2}{(w-s)d^2}\frac{ds}{dt} \quad , \tag{9}$$

and a differential equation for $s(t)$:

$$\frac{ds}{dt}\Big(s\mu_1 + (w-s)\mu_2 \Big) = p_0 d^2 \quad . \tag{10}$$

Equation (10) shows that the interface speed ds/dt begins very fast ($p_0 d^2/w\mu_2$) and slows down continually until pinchoff (when the speed is lowest, at $p_0 d^2/w\mu_1$).

Integrating (10) with respect to t, then using $s = 0$ when $t = 0$, and solving the subsequent quadratic equation, gives

$$s(t) = \frac{w\alpha}{1-\alpha}\left(-1 + \left[1 + t\frac{2p_0 d^2}{w^2\mu_1}\left(\frac{1-\alpha}{\alpha^2}\right)\right]^{1/2} \right) \quad , \tag{11}$$

where $\alpha = \mu_2/\mu_1 = 5 \times 10^{-5}$ is the ratio of viscosities. From this

$$\frac{ds}{dt} = \frac{p_0 d^2}{w\mu_2}\left[1 + t\frac{2p_0 d^2}{w^2\mu_1\alpha^2}(1-\alpha)\right]^{-1/2} \quad , \tag{12}$$

The expressions (11, 12) can be substituted into (8, 9) to get formulas for $\lambda_1(t)$ and $\lambda_2(t)$.

A more useful treatment of (10) is to integrate from $s = 0$ to pinchoff at $s = w$, at a time $t = t_p$. This calculation gives the *pinchoff time:*

$$t_p = \left(\frac{w}{d}\right)^2 \frac{\mu_1}{2p_0}(1+\alpha) \quad . \tag{13}$$

In terms of t_p the above formulas can be written more compactly. The *position of the nose* is

$$s(t) = \frac{w\alpha}{1-\alpha}\left(-1 + \left[1 + \frac{t}{t_p}\left(\frac{1-\alpha^2}{\alpha^2}\right)\right]^{1/2} \right) \quad , \tag{14}$$

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and the *speed of the nose* is

$$\frac{ds}{dt} = \frac{p_0 d^2}{w\mu_2}\left[1 + \frac{t}{t_p}\left(\frac{1-\alpha^2}{\alpha^2}\right)\right]^{-1/2} \quad . \tag{15}$$

The above formulas are more handy for obtaining $\lambda_1(t)$ and the relatively small $\lambda_2(t)$. A conclusion from that is that the spatial pressure *gradient* is tiny in the core fluid (where the fluid pressure p_2 stays close to zero), in contrast with huge pressure gradients in the more viscous insulating fluid.

Numerical estimate of the time to blow and the decline of electrical current: If the length scale d is about the same as the width of viscous fluid on the wall then $w/d = 0.5 \times 0.35/(0.06) = 3$, and if $\mu_1 = 10$, $\mu_2 = 5 \times 10^{-4}$, and if $p_0 = 10^9$, then equation (13) tells us that $t_p = 40$ nanoseconds. This compares with the measurements of 40 nanoseconds for the duration of the peak in current, and 500 nanoseconds measured for a fuse to completely blow correctly. However, we are uncertain about the correct choice of values for the length-scale d, and for the pressure p_0 associated with driving the thin layer of viscous flow. We have assumed that p_0 and d are constants, but it may be that one should account for the possibility that d increases from zero and p_0 declines.The estimate of t_p is most sensitive to the choice of d.

The electrical resistance of this part of the length of fuse, increases in direct proportion to the ratio of the original cross-sectional area of the conducting pathway to the cross-sectional area of the neck. For a square cross-section this is $4w^2/(4(w-s)^2)$. So if R_0 is the original resistance and this changes to R then we have

$$\frac{R}{R_0} = \frac{w^2}{(w-s)^2} \quad . \tag{16}$$

A mathematical model could be made for the current due to three resistors in series, two fixed resistors on either side of the pinchoff, and one increasing resistance to describe the pinchoff section. Instead we re-express (16) just naively in terms of currents to get

$$\frac{I}{I_0} = \frac{(w-s)^2}{w^2} = \left[1 + \frac{\alpha}{1-\alpha}\left(1 - \left[1 + \frac{t}{t_p}\left(\frac{1-\alpha^2}{\alpha^2}\right)\right]^{1/2}\right)\right]^2 \quad . \tag{17}$$

The right-hand side of (17) decreases from 1 to zero, as t increases from 0 to the pinchoff time t_p. The shape of the plot of current as a function of time decreases in a way which could be compared with the measurements. Preliminary sketches suggest that when $\alpha = 5 \times 10^{-4}$ the current falls initial sharply from $I/I_0 = 1$ at $t = 0$, then $I/I_0 = 0.5$ when $t = 0.2t_p$ and then I/I_0 tails down to zero at $t = t_p$.

3(b) List of full model contents:

A more detailed model would contain:

(M1) A fully three-dimensional description of the dog-bone geometry of the fuse.

(M2) A model of the high Reynolds number flow in the low-viscosity melted core, solved simultaneously with equations for a low Reynolds number flow of the high viscosity thin layer of insulator. The interaction across the moving interface between the two fluids is likely to be by the normal stress (pressure) distribution alone. The pressure varies with position and depends on the velocities of the two fluids.

(M3) The fluid dynamics of (M2) needs a description of the electron drift force (modelled as a body force).

(M4) A pair of incompressible (constant density) fluids may be right, but if the compressibility of the fluids were wanted then some constitutive relation for each liquid $p = p(\rho)$ would be need between the pressure and the density, as well as the mass continuity equation which in full is:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0 \quad , \tag{18}$$

where **u** is the fluid velocity vector field.
(M5) The melted conductor contains either many bubbles (which could be described as one constituent of a single continuum with compressible-flow properties), or a single bubble, growing in size with a free surface of constant (low) pressure to be tracked up to the time of resolidification.
(M6) To account for the debris seen in a blown fuse a better understanding of the re-solidification process might be needed. And to do that one would need to model the temperature field in space and time, and follow the densities of the elastically compressed media after solidification.

4. Thoughts on Further Work:

4.1: A detailed model of the Joule heating process is probably not needed as there is a plentiful supply of energy to melt the electrically conducting core. Of more interest is the later stages when the heat is melting the surrounding insulator. Are there places, such as the fuse corners where more melting occurs? The cooling process was not discussed but if that were modelled it may help to explain the presence, size and shape of the cavity and the debris all around the fuse.

4.2: The fluid dynamics is complicated. The boundary of the low-viscosity core is moving in response to the low pressure in the core. The flow through the gap during pinchoff is another low-pressure region, and this may promote the closure of the insulator. On the other hand there is an enormous body force acting downwards on the flow, and it is not clear how the core responds to this. Do the bubbles have a chance to move up to make a cavity, or do they vanish on re-solidification? If this is to be investigated then a two-phase model of the fluid medium would be needed – a difficult matter. More simply, the fluid may rupture near the place of pinchoff and this might promote the growth of the cavity as one entity in the region above the pinchoff. To investigate this a good model would be needed of the fluid flow (especially the pressure) through the gap during pinchoff.

4.3: The observed needle growth occurs under conditions of sustained voltage difference, and heating at 125°C, for 1000 hours. What physics describes the accretion of matter at the needle tip? This is better known in tin where 'whiskers' can grow. See Liu, Chen, Liu and Chou (2004).

4.4: Can the fuse be designed to promote pinchoff at a desired position? Could the conductor be made with a narrow section at its midpoint where pre-existing noses of silicon dioxide could readily begin pinchoff? This issue is related to the possibility that in small fuses the melting may be greatest at the ends of the fuse at the corners of the silicon dioxide region.

References:

Liu, S. H., C. Chen, P. C. Liu and T. Chou (2004) Tin whisker growth driven by electrical currents. *Journal of Applied Physics*, **95**, No. 12, 7742–7747.

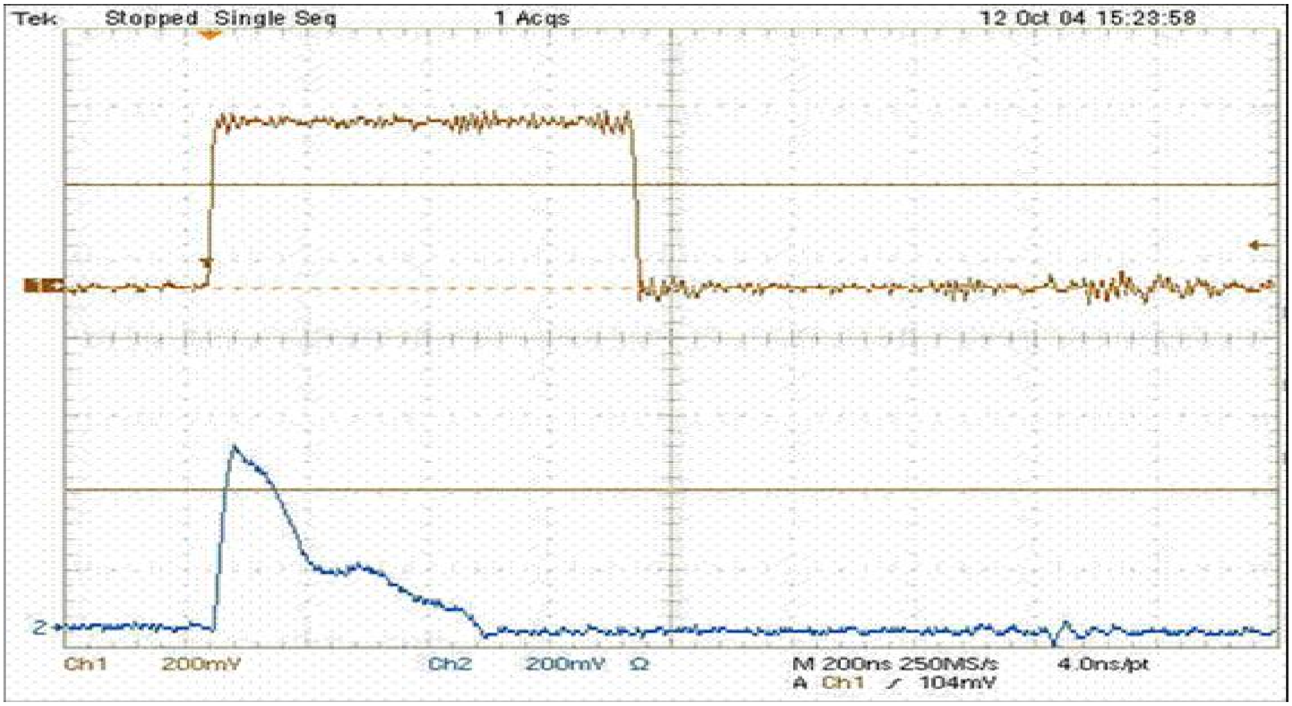

Figure 1: Upper plot: voltage across the fuse. Lower plot: the current $I(t)$ through the fuse as a function of time t after the voltage is first applied across the fuse. The initial current rise, to a peak of about 90 milliamps, is in a time of 1 nanosecond or less (it is quicker than the instrument could measure). The width of the upper half of the peak in current is about 40 nanoseconds. The whole blowing process is completed in 500 nanoseconds. [Courtesy *Analog Devices*.]

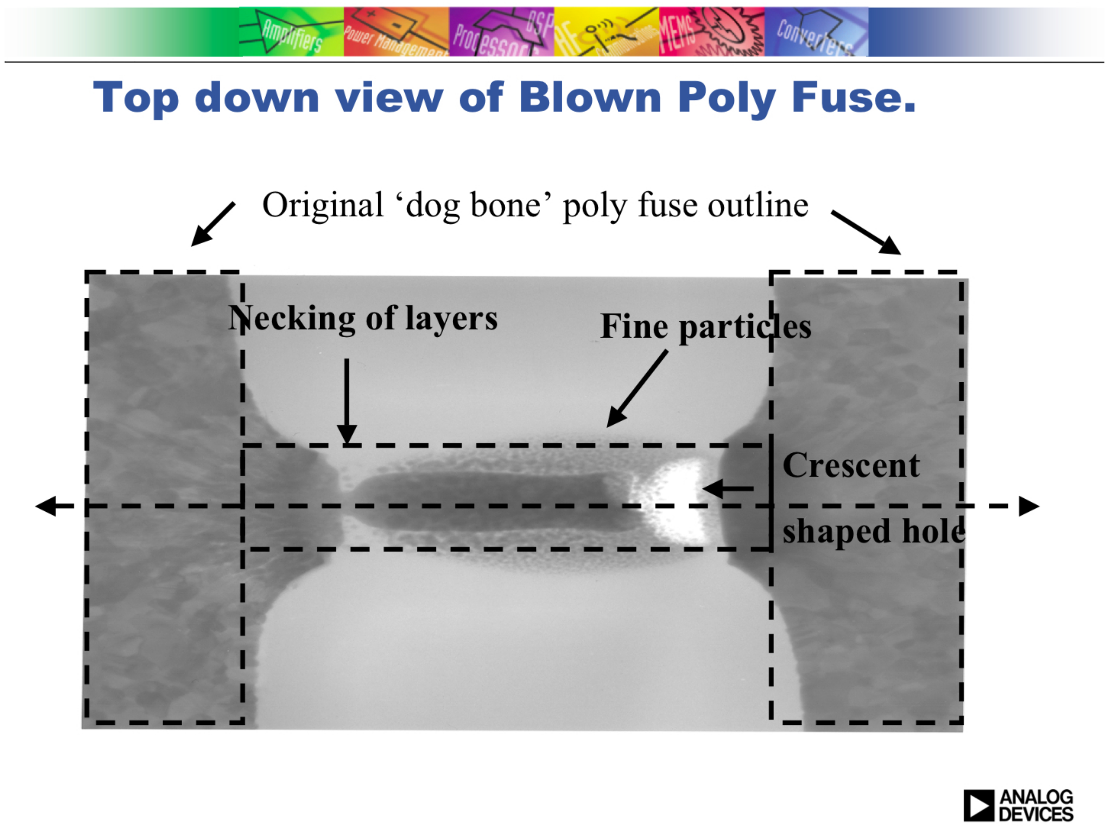

Figure 2: The geometry of the small fuse before and after it has blown. Before blowing, the central part of the dogbone is 1.5 micometres by 0.35 micrometres. The dark material is conducting tungsten silicide with polysilicon beneath it. The lighter material is electrically insulating silicon dioxide which surrounds the conductor. Note the mixture of materials shown as a haze of granules, and the white patch which is the cavity near the 0 volts end; (the +6volts end is at the left in this image). [Courtesy *Analog Devices*.]

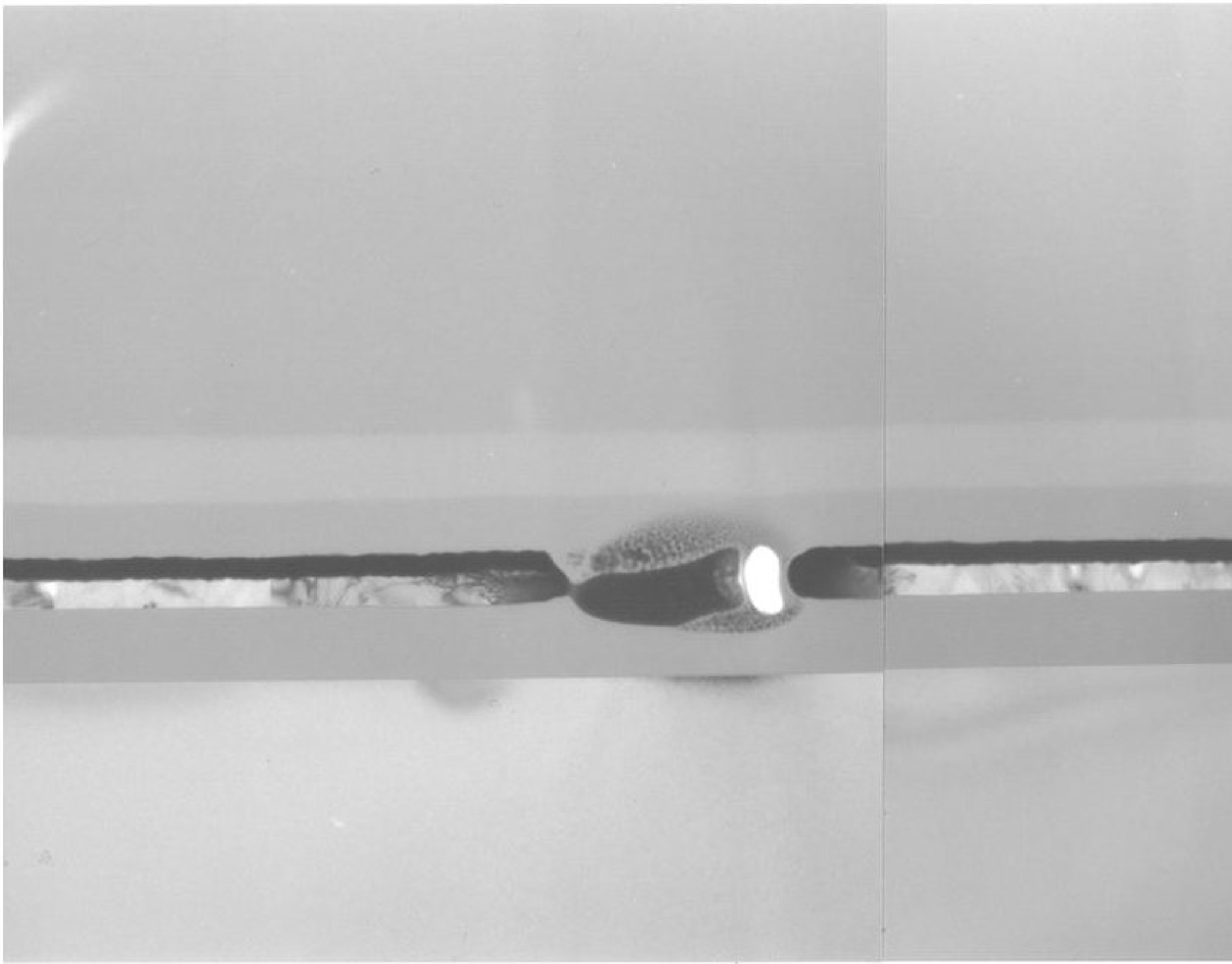

Figure 3: As figure 2. Side view showing the layer of tungsten silicide above (dark) and the polysilicide below to which the tungsten silicide is attached. Examples of necking (incomplete pinchoff?) at the left and the pinchoff by insulating material around the white cavity at the right, are clearly shown. [Courtesy *Analog Devices*.]